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Regional Development under Stagnation

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A. Soft Models of Herman Wold's Type: Models of Environmental Problems and Underdevelopment

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I. Introduction[1]

The analysis of regional stagnation is related to many problems which usually have been dealt with separately in different sciences. There is an obvious lack of a well developed and commonly approved theory covering the whole field of regional analysis. We are able to imagine many different complex models, but there are only rough theoretical guide lines in outlining the relations and variables which ought to be included. Often the variables of theoretical interest (e. g. regional attractiveness) cannot be observed directly, so one has to rely on indicator variables which are assumed to have some degree of association with them.

One answer to the problem of modeling complex models was given by the »path analysis« approach, well known in social sciences (cf. Blalock, 1971). Path models are based on structural equation systems which may be recursive or interdependent, thus complicated relations among the model variables can be investigated. Recent developments of this approach provide estimation procedures comprising »latent variables« which are observed by a set of indirect indicator variables (cf. Jöreskog and Sörbom, 1977). However, this conventional method of path analysis and its extension to latent variables relies upon exact knowledge about the theoretical structure of the model and about the statistical distribution properties of the variables. With regard to the missing knowledge in most of the subjects of complex regional analysis, a statistically less pretentious method is called for.

In the first part of this paper a new approach developed by Herman Wold will be presented which provides the possibility of a complex path analysis on the base of less knowledge-intensive assumptions. Using the partial least squares method in

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1 The author wishes to express his sincere appreciation and thanks to Professor Herman Wold as instigator of his studies on the partial least squares approach. He is especially indebted to Professor Wold for giving him the opportunity to spend the academic year 1977 together with him at the Universities of Philadelphia, Seattle and Stanford. This research visit was financed by the Stiftung Volkswagenwerk, Germany. A large part of the computer work was carried out at Wharton School, Philadelphia. Preparatory research took place at Frankfurt University and was financed by the Deutsche Forschungsgemeinschaft, Germany. In Frankfurt, too, the final investigations were carried out. Finally, the author acknowledges with gratitude the support of Professor Werner Meissner in the provision of facilities and fruitful comment; without him this work would not have been possible.

estimating the model parameters we have a distribution-free and data oriented approach aiming at exploratory analysis.
In the second part three examples are discussed. A pilot study of a regional environmental model shows the difficulty of exploring stable model relations on the base of poor environmental and insufficient regional policy data. The investigation of Adelman/Morris on different growth patterns among different subgroups of underdeveloped countries is regional analysis on a world wide scale. It can easily be transfered to groups of districts or counties of one country. In the third example Wold's »soft« method is compared with the conventional »hard« approach of path analysis. The examples illustrate the flexibility of Wold's »soft models«, using time series, cross-sectional and panel data.

II. The Partial Least Squares Approach

Herman Wold has developed the partial least squares (PLS) approach for estimating multi-scaled path models (cf. Wold, 1977). The term »soft modeling« indicates the intermediate position of his method between descriptive and inferential statistics. The application of simple correlation techniques allows the analysis of complex and interdisciplinary constructs for which a priori knowledge is scarce. The main difference between Wold's method and the classical path model approach is its ability to deal with unobserved multi-scaled latent variables in a causal stream of the path model. On the basis of a factor analysis background, Karl G. Jöreskog has developed independently of Herman Wold a quite different approach for estimating multi-scaled path models (cf. Jöreskog and Sörbom, 1977). Jöreskog's LISREL method (Linear Structural Relation System) is a very general computer program for estimating the unknown coefficients in a set of linear structural equations. Since he uses the maximum likelihood estimation method, he requires exact knowledge about the distribution of observables and about the detailed specification of the theoretical model. Parameters can be fixed, free or constrained, and the full range of inferential statistics is available. Nevertheless, only the specification of the model can be seriously hampered by identification problems, and if there are only weak hypotheses about the theoretical structure of the model, Jöreskog's sophisticated LISREL method might be a pitfall for the analyst. Because of their different statistical assumption backgrounds, the two approaches are complementary rather than competitive. In socio-economic and other interdisciplinary areas, the simplicity of the partial least squares approach is its strength.

The basic idea of partial least squares modeling was developed by Herman Wold in 1966 when he discovered an iterative regression procedure for calculating the principal components and the canonical correlations. Let X be a $N \times K$ matrix of observations (K variables and N case values),

$a' = (a_1, \ldots, a_K)$ a K-element coefficient vector,
$l' = (l_1, \ldots, l_N)$ a N-element vector.

Let l be a linear form in X

$$l = Xa$$

The principal component (PC) approach is to find such a vector l that var (l) is at a maximum under the constraint $a'a = 1$. This condition gives $(X'X)a = \lambda a$; $l'l = \lambda$. Here the vector l is called the first principal component. The classical solution for a and λ is derived using the eigenvalue approach (cf. Johnston, 1972, p. 322).
Herman Wold solves this problem iteratively:
Standardize the K vectors x_k of X;
starting with $a^1 = (1, \ldots, 1)$ gives $l^1 = Xa^1$;
then by iteration:

(i) estimate a^i by simple regression (l^i given from iteration step $i-1$; e^i_k N-element residual vector):

$$x_k = a^i_k l^i + e^i_k \qquad [1]$$

(ii) define l^{i+1} as

$$l^{i+1} = \lambda^i X a^i \quad \text{with} \quad \lambda^i = 1/\mathrm{var}(Xa^i) \qquad [2]$$

repeating equations [1] and [2] for $i = i + 1$ and so on.
This gives l^i which converges against l, the first principal component, and a^i which converges against a, the first eigenvector, both of which are numerically equivalent to the result obtained using the classical approach.
The canonical correlation (CC): in this case there are two matrices of observations $X_{1(N\times K)}$ and $X_{2(N\times H)}$. Two linear forms are sought

$$l_1 = X_1 w_1$$

$$l_2 = X_2 w_2$$

(l_1, l_2 N-element, w_1 K-element and w_2 H-element vectors) whose correlation $r(l_1, l_2)$ is maximal (cf. Johnston, 1972, p. 321).

The partial least squares (PLS) procedure:
by the standardization of
x_{1k}, as N-element vector of X_1; $k = 1, \ldots, K$,
and
x_{2h}, as N-element vector of X_2; $h = 1, \ldots, H$,
starting with $w^1 = 1$ unit K-element vector and
$w_2^1 = 1$ unit H-element vector, we get the initial linear forms:

$$l_1^1 = x_1 w_1^1 \; ; \qquad l_2^1 = x_2 w_2^1$$

Then by iteration:

(i) apply multiple regression to obtain w_1^i; the term l_2^i is given from the previous iteration, while e_1^i is an N-element residual vector:

$$l_2^i = x_1 w_1^i + e_1^i \qquad [3]$$

(ii) define the linear form l_1^{i+1} as

$$l_1^{i+1} = \lambda_1^i x_1 w_1^i \quad \text{with} \quad \lambda_1^i = 1/\mathrm{var}(x_1 w_1^i) \qquad [4]$$

(iii) apply multiple regression to obtain w_2^i, using l_1^{i+1} from equation [4]:

$$l_1^{i+1} = x_2 w_2^i + e_2^i \qquad [5]$$

(iv) define the linear form l_2^{i+1} as

$$l_2^{i+1} = \lambda_2^i x_2 w_2^i \quad \text{with} \quad \lambda_2^i = 1/\mathrm{var}(x_2 w_2^i) \qquad [6]$$

repeating stages (i)–(iv) for $i = i+1$ and so on.

(v) use simple regression to obtain the canonical correlation coefficient r. After convergence,
$l_1^i \rightarrow l_1$ and $l_2^i \rightarrow l_2$, the regression

$$l_2 = r l_1 + d \qquad [7]$$

can be performed, where d is an N-element residual vector.

The partial least squares approach can also be represented graphically. The latent variable l is enclosed in a circle, the observed variables X are represented by rectangles. Simple regressions (so called outward-directed weight relations) are indicated by single arrows, multiple regressions (so called inward-directed weight relations) are indicated by bundled arrows.

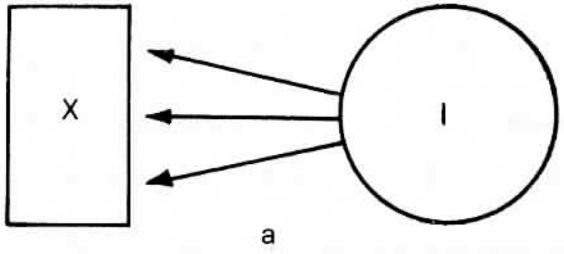

Figure 1: The Principal Component Model

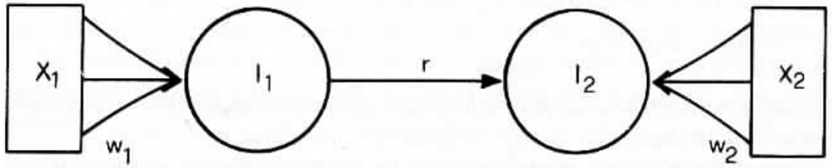

Figure 2: The Canonical Correlation Model

In the light of the iteration procedure, the canonical correlation approach can be regarded as a combination of two optimization principles:
(i) maximization of the correlation between latent variables,
(ii) minimization of the residuals in forming the weights w.
The use of these principles leads to an amalgamation of the principal component and the canonical correlation approach in the following model:

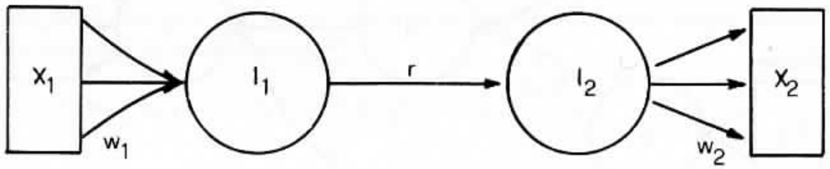

Figure 3: The Hybride Model of the Principal Component Approach and the Canonical Correlation Approach

Using the preceding notation and dropping the iteration index, one obtains the corresponding equation system in accordance with the two principles:

(i) apply multiple regression:

$$l_2 = X_1 w_1 + e_1 \qquad [3']$$

(ii) define:

$$l_1 = \lambda_1 X_1 w_1 \qquad [4']$$

(iii) apply simple regression:

$$x_{2h} = w_{2h} l_1 + e_{2h} \; ; \quad h = 1, \ldots, H \qquad [5']$$

(iv) define:

$$l_2 = \lambda_2 X_2 w_2 \qquad [6']$$

(v) apply simple regression:

$$l_2 = r l_1 + d \qquad [7']$$

It has been shown that this procedure always converges and that the two principles described are equivalent (one is sufficient) (cf. Areskoug et al., 1975).

Herman Wold's main generalization of the iterative principle component and canonical correlation techniques consists of the introduction of more than two latent variables. A straightforward development of the preceding equation scheme allows the estimation of arbitrarily complex multi-scaled path models. To illustrate the general procedure, consider the following model:

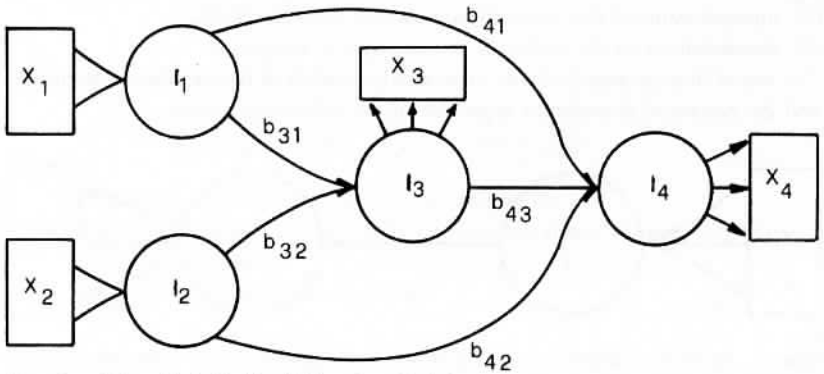

Figure 4: A Four-block Model with Two Inner Relations

Let $\mathcal{L} = [l_1, \ldots, l_4]$, $\mathcal{E} = [e_1, \ldots, e_4]$ and B a subdiagonal matrix 4×4:

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b_{31} & b_{32} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & 0 \end{bmatrix}$$

Then the linear equation system

$$\mathcal{L} = B\mathcal{L} + \mathcal{E}$$

describes the preceding path model (or structural model) of latent variables. This is the generalization of equation [7]. To measure the latent variables l_i, Herman Wold had to generalize the equations [3]–[6]. Obviously, the crucial step is in equations [3] and [5]. The definition equations [2] and [4] are still valid. How can the influence of the other latent variables of the model be combined? The simplest rule is to add those latent variables which are connected with arrows to the variable in question. For instance, the »first block« gives rise to the multiple regression equation:

$$l_3 + l_4 = X_1 w_1 + e_1 \qquad \text{(corresponding to equation [3])}$$

To avoid cancelling of the left side terms, sign factors $s_{ik} = \text{sign } r(l_i, l_k)$ are introduced:

$$s_{31} l_3 + s_{41} l_4 = X_1 w_1 + e_1 \; .$$

In Wold's terminology this is an inward-directed weight relation. The combination rule should work in the direction of higher correlation between connected latent variables, but it is not the result of an analytical optimization criterion. In the third block of the model one has on outward-directed weight relation (corresponding to equations [5']):

$$x_{3j} = w_{3j}(s_{13} l_1 + s_{23} l_3 + s_{43} l_4) + e_{3j} \; ; \quad j = 1, \ldots, J$$

The full equation scheme of a general partial least squares basic design is given in the appendix of this paper.

The partial least squares procedure can be divided into two models: the structural path model and the measurement model (this partitioning is not used by Wold; it follows the LISREL presentation by Jöreskog). The partial least squares approach starts with the measurement of the latent variables. In a second step, the linear equation system of the structural path model can be estimated with the help of the single equation partial least squares approach in the case of a recursive system (for non-recursive systems cf. Hui, 1978).
Specification problems in the partial least squares approach: In addition to the lack of an analytical optimization rule for combining the influence of latent variables in the weight relation there is another problem with the measurement model. There should be a general rule for choosing the inward- versus outward-directed relation in determining the weight relations. Herman Wold interpreted the inward-directed relations as causal and the outward-directed relations as effect indicators. If the weight relations [3′] and [5′] are examined, this seems to be a plausible explanation, bearing in mind that in the inward-directed case the observables are the independent variables and in the outward-directed case they are the dependent ones. However, if the definition equations [4′] and [6′] are studied, the picture changes. Another explanation is given with the notion of »reflective« versus »formative« indicators (cf. Wold, 1979, p. 12). The inward-directed relation will result in higher correlations between the latent variables because of the »canonical correlation effect« (cf. p. 271). If the model builder knows how to interpret this higher correlation, he has obtained a further rule.
In December 1977 Herman Wold introduced a third kind of coefficient set, in addition to the inner and the weight relations. When the latent variables have been estimated, simple regressions can be performed between the latent variables and the corresponding observables:

$$x_{kj} = p_{kj} l_k + \gamma_{kj} \, ; \quad k = 1, \ldots, K; \; j = 1, \ldots, J_K$$

Because of the standardization of the observables x_{kj} and the latent variables l_k, those p_{kj} are at the same time correlation coefficients. They are known as the block structure of the model (cf. Wold, 1979).
To repeat: we have the problem of choosing the relational direction of each block and we have to choose between weight relations or block structure for the purpose of interpreting the observed indicators. To give a simple application rule, in my experience, the model type with only outward-directed relations (A mode) is the most stable one with respect to certain properties, and the block structure represents clear-cut correlation coefficients.
Open questions: The convergency of the partial least squares procedure has been shown for the one and two block models (cf. Areskoug et al., 1975), but not for

higher-order models. In most applications convergency was no problem. The models converge in most cases with less than 10 iterations.
The properties of the partial least squares estimates have not yet been fully explored. Assuming the existence of the block structure, the partial least squares estimates are consistent at large (cf. Wold, 1979).
The equivalence of the two optimization principles in the case of two-block models does not seem to hold for higher-order models.
Because of the partial correlations (correlations with a partial set of the sample holding fixed the rest of the sample) it cannot be expected that the procedure converges against a global optimum.
Further aspects: Corresponding to the higher components in the principle component and the canonical correlation approach, higher »dimensions« can be estimated in the partial least squares procedure by using the residuals of the block structure as new »observables« (cf. Wold, 1966). A prediction-oriented significance test for these dimensions is available.

III. Examples of the Application of the Partial Least Squares Approach

1. A Socio-economic Environmental Model for the State of Hessen

A quantitative approach for researching environmental economic questions has the following limitations:
(a) theoretical knowledge is scarce,
(b) the statistical recording of data is incomplete and inadequate.
Thus the following investigation is seen only as a pilot study, investigating in what way the economic situation, the environmental situation and the governmental activity of a region are interdependent. Of course, these three general features are vague concepts and they cannot be measured directly. Therefore, a path model with latent variables (cf. Figure 5) would seem to offer an adequate description (cf. Apel et al., 1978, p. 299 f.).
Data Problems: The observational interval is very short (1962–1973). There are high correlations between all variables. The observables of the ecological situation and of governmental activity do not indicate exactly what they ought to. For instance, the ecological variables are more an expression of the emission situation, which is not directly related to the real extent of pollution. The governmental variables are expenditure-oriented; they ought to be more effect-oriented.

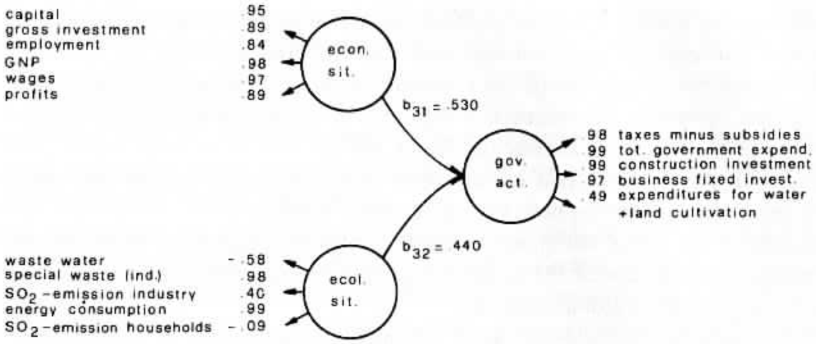

Figure 5: Variables and Results of the Basic Model

Our hypothesis of a low level of influence of the ecological situation on governmental activity was confirmed. During the period for which data was collected, public discussion of environmental problems and governmental activity was just beginning. The negative sign of the waste water variable is plausible because the amount of untreated water declined dramatically in the late 1960's. However, sensitivity testing is needed in order to provide conclusive evidence of the validity of the model.

Table 1 shows comparisons between models estimated with different relation directions and different weight schemes. The notation is explained in the appendix (weight scheme 0 = relation oriented; 1 = block oriented).

Mode	A	C	A		C	
Weight scheme	1	1	0	1	0	1
Iterations	4	6	5	3	11	9
b_{21}	-	-	.964	.963	.994	.992
b_{31}	.530	.690	.719	.623	1.077	.783
b_{32}	.440	.310	.239	.339	-.084	.212

Table 1: Inner Relations Comparison

In view of the low number of case values (18–19 estimates related to only 12 observation points), the instabilities are not surprising. The inner relations of the C mode change much more than those of the A mode. This is apparently due to the high correlation between the economic and ecological situations ($r_{21} = b_{21}$), both of which are independent variables of the inner relation regression. The inward related direction in the A model (canonical correlation case) provides for higher correlations between latent variables; it binds them more closely together than might be desired. Nevertheless, the stronger influence of the economic situation variable seems to be a consistent pattern.

2. A Socio-economic Partial Least Squares Model of the Growth Process of Underdeveloped Countries

In 1967 Adelman and Morris carried out what has since become a famous factor analytical study concerning the interdependence of economic growth and socio-political change in developing countries. For this study they gathered cross-sectional data from 74 countries over the period 1957–1962, for a total of 43 variables, most of which were qualitative (ordinally scaled). In 1975 when Irma Adelman worked together with Herman Wold she proposed the following partial least squares model related to her studies:

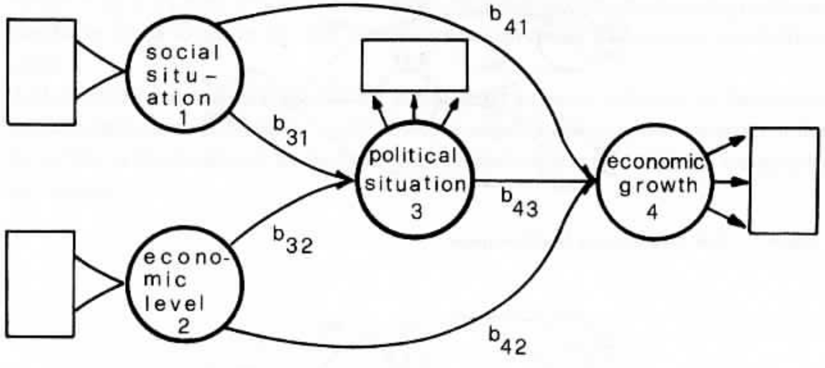

Figure 6: Irma Adelman Model

At this time, a general computer program did not exist; only preliminary investigations of a model with three latent variables could be conducted (cf. Wold, 1975). The author does not wish to reanalyze the data generated by Adelman/Morris, nor does he intend to present a detailed picture of partial least squares estimation results. However, it may be interesting to try to describe certain properties of the partial least squares approach through the analysis of such a relatively complex model. The fact that the cross-sectional sample is divided into three subsamples makes this model an especially appropriate subject for such an investigation. There are 28 low level countries; here economic activity tends to be embedded in traditional social organization and values. In 21 intermediate level countries, although the spread of the market economy has developed relatively well, economic growth is still seriously impeded by the uneven pace of social development and the influence of government. A further 25 countries represent a high level, where socio-structural bottlenecks are no longer significant barriers to economic and political measures aimed at promoting development. At the lowest level, characteristic of sub-Saharan Africa, Adelman and Morris find that social forces are typically the most important non-economic influence upon economic activity, with political change, on the whole, exercising a negligible impact. By contrast, at the highest of the three levels studied, it is political forces that are crucial to economic performance, while social influences have little systematic effect. Economic forces are important throughout but it is only at the highest level of development that they assume their full significance (cf. Adelman and Morris, 1967, p. 7).

The following partial least squares estimation results (A mode) would appear to demonstrate quantitatively the Adelman/Morris findings:

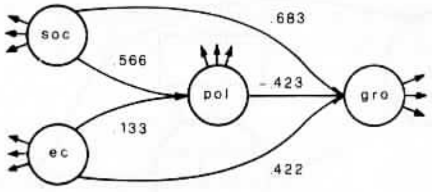

Figure 7: Low Development-level Countries

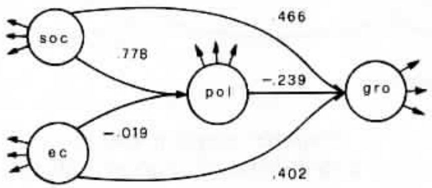

Figure 8: Intermediate Development-level Countries

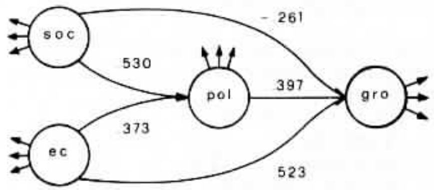

Figure 9: High Development-level Countries

Table 2 presents the variable list which is identical for each model, while in Table 3 can be found the corresponding loadings of the first block of the models, comparisons of two weight schemes and results for reduced samples. Again, except for the total sample, the number of estimates is larger than the number of case values per variable. For this reason, the number of observables has been reduced in order to test the stability of the inner relations. Variables with low weights were cancelled. In the case of the low sample, the reduction leads to very similar results. In the case of the high sample, there is a remarkable parameter shifting (given in the last two columns of Table 3). Another more general problem for multi-scaled path models is the question of what kind of observables constitute an adequate latent variable. For instance, some of the economic level indicators can be regarded as social indicators and vice-versa. Model building is an open process and is based on theoretical and subjective assumptions.

If the B or C modes of the total or subsample models are estimated, problems similar to those in the model for Hessen are encountered and serious instabilities arise.

Although we have not discussed the partial least squares estimates in detail, the initial results seem promising as a first step toward finding regular patterns in the data. The procedure leads to very complex results with a rather simple estimation technique.

1. Block of social indicators:

1 size of traditional agricultural sector
2 modernization of outlook
3 size of middle-class
4 extent of social mobility
5 extent of literacy
6 extent of masscommunication
7 degree of social tension
8 national integration
9 ethnic homogeneity
10 crude fertility rate
11 urbanization
12 agricultural organization
13 social organization

2. Block of economic level indicators:

1 level of infrastructure
2 level of industry
3 level of agriculture
4 dualism
5 per capita gross national product
6 structure of trade
7 abundance of natural resources

3. Block of political conditions indicators:

1 degree of administrative efficiency
2 degree of centralization of political power
3 democratic tradition
4 degree of freedom of the press
5 predominant basis of the political party system
6 factionalization of parties
7 extent of political stability
8 extent of leadership commitment
9 strength of labor movements
10 political strength of the traditional elite
11 political strength of the armed forces

4. Block of economic growth indicators:

1 rate of change in per capita gross national product
2 investment rate
3 improvement tax
4 change in degree of industrialization
5 improvement in financial institutions
6 improvement in agriculture
7 improvement in infrastructure
8 rate of improvement of human resources

Note: A detailed description of the variables may be found in the Adelman/Morris books.

Table 2: Variable List of the Adelman Model

sample	total		high development level		intermediate development level		low development level		reduced high		reduced low	
weight scheme	0	1	0	1	0	1	0	1	0	1	0	1
cases N:	74		25		21		28		25		28	
loads of	w	p	w	p	w	p	w	p	w	p	w	p
1. block												
1	-.11	-.92	-.14	-.80	-.18	-.59	-.16	-.63		-.83		-.61
2	.11	.85	.16	.82	.28	.66	.22	.79		.84		.79
3	.12	.87	.14	.88	.11	.36	.26	.74		.87		.77
4	.12	.93	.17	.83	.16	.65	.26	.83		.83		.86
5	.10	.90	.10	.63	.20	.61	.14	.56		–		.55
6	.11	.90	.13	.66	.10	.46	.17	.37		.66		.34
7	-.03	-.31	-.04	-.27	-.12	-.47	.01	.25		–		–
8	.09	.84	.08	.53	-.13	-.15	-.02	-.14		–		–
9	.05	.53	-.04	.13	-.20	-.37	-.07	-.22		–		–
10	-.07	-.64	-.10	-.73	-.15	-.43	.02	.28		–		–
11	.10	.85	.11	.59	.17	.65	.09	.43		–		–
12	.11	.84	.17	.76	.10	.50	.17	.65		.79		.63
13	.09	.84	.05	.36	-.00	-.11	.02	.28		–		–
inner relations:												
b_{31}	.11	.17	.59	.53	.92	.78	.65	.57	.51	.45	.61	.57
b_{32}	.60	.55	.31	.37	-.18	-.02	.02	.13	.36	.41	.07	.13
b_{41}	.38	.39	-.50	-.26	.39	.47	.70	.68	-.01	.10	.84	.82
b_{42}	.55	.54	.61	.52	.49	.40	.42	.42	.73	.59	.35	.35
b_{43}	-.16	-.15	.50	.40	-.38	-.24	-.45	-.42	-.10	-.04	-.55	-.53
correlations:												
r_{21}	.91	.91	.91	.91	.59	.65	.76	.76	.94	.93	.74	.75
r_{31}	.66	.67	.87	.87	.81	.77	.67	.67	.84	.84	.67	.66
r_{32}	.70	.71	.85	.86	.36	.49	.51	.56	.83	.84	.53	.56
r_{41}	.77	.78	.49	.56	.37	.55	.72	.72	.59	.62	.73	.73
r_{42}	.78	.78	.58	.63	.58	.59	.72	.70	.64	.65	.69	.67
r_{43}	.47	.49	.58	.62	.12	.32	.23	.27	.50	.54	.19	.21

Note:
0 = relation oriented weight scheme
1 = block oriented weight scheme
w = weight relations
p = block structure

Table 3: Estimates of the Adelman Models (Estimated in Mode A with Two Different Weight Schemes)

3. A Longitudinal Model Estimated by LISREL and Partial Least Squares Approach

In the LISREL literature the following longitudinal model of alienation is often used as an example (cf. Jöreskog et al., 1977, 1978):

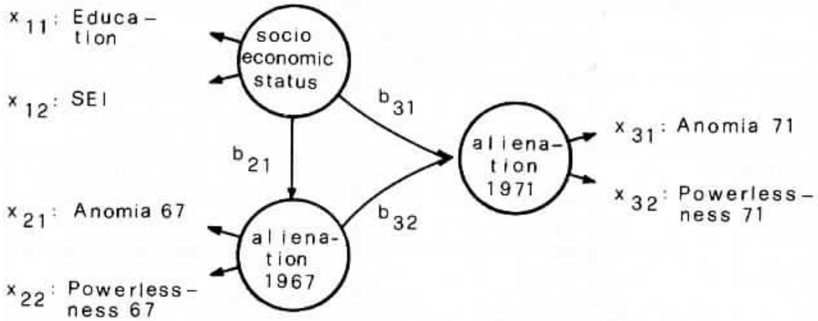

Figure 10: LISREL Alienation Model

Socio-economic status (SES) is a background variable and is represented by a subject's education (years of schooling completed), together with Duncan's Socio-economic Index (SEI). Alienation is measured with the Anomia subscale and the Powerlessness subscale (in 1967 and 1971).

	x_{11}	x_{12}	x_{21}	x_{22}	x_{31}	x_{32}
x_{11}	1.					
x_{12}	.54	1.				
x_{21}	- .36	- .30	1.0			
x_{22}	- .41	- .29	.66	1.		
x_{31}	- .35	- .28	.56	.47	1.	
x_{32}	- .37	- .28	.44	.52	.67	1.

Table 4: Correlation Matrix of the Observables: N=932 (cf. Jöreskog et al., 1978, p. 28)

Because of the fixed parameters $p_{11} = p_{21} = p_{31} = 1$ in the LISREL specification, and because of the standardization in the partial least squares approach, the results are comparable only in a restricted sense. The outer relations are not comparable; the inner relations may be slightly different because of the different optimization conditions (restricted versus non-restricted). Nevertheless, the inner relations of the partial least squares approach emerge as very similar to the corresponding estimates of LISREL. Aware of the vagueness of the »alienation« and »socio-eco-

	LISREL		partial least square	partial least square
	1A	1B[1]	mode A	mode C[2]
inner relation				
b_{21}	- .641	- .575	- .432	- .432
b_{31}	- .174	- .227	- .180	- .182
b_{32}	.705	.607	.520	.518
outer relation				
p_{11}	1.	1.	.909	.949
p_{12}	5.329	4.846	.841	.742
p_{21}	1.	1.	.910	.910
p_{22}	.889	.979	.913	.914
p_{31}	1.	1.	.918	.917
p_{32}	.849	.922	.909	.910
correlation				
r_{21}			- .432	- .438
r_{31}			- .404	- .409
r_{32}			.598	.598
χ^2	71.47	4.73		
d.f.	6	4		

Note:
1) Cf. Jöreskog et al., 1978, p. 30. The column 1A of the LISREL specification assumes uncorrelated residuals for the observables. This manner of specification is directly comparable with the specification of the partial least square model. In column 1B correlations (covariances) between the residuals of the corresponding alienation indicators are introduced. Such refinement is not possible in the partial least square approach.
2) Only the first block with inward-directed weight relations.

Table 5: Comparison of Estimation Results

nomic status« concepts, the analyst should not place too much emphasis upon the numerical size of these parameters. The shifting in the parameter values from the statistically insignificant model 1A to the significant one 1B (χ^2-test) is too small to be interpreted as a representation of distinguishable phenomena. So far, LISREL 1A, 1B and the partial least squares model indicate the same pattern. This seems a promising result for the latter procedure.

IV. Appendix: Basic Design of a Partial Least Squares Model with K Latent Variables

1. Notation

Be K: number of latent variables of the model,

L: number of blocks of observables (L < K),

$\mathcal{L} = [l_1,\ldots,l_K]$; l_k (N-element vector) k-th latent variable,

$B = (b_{kj})$ a $K \times K$ subdiagional matrix of inner relation coefficients,

$\mathcal{E} = [e_1,\ldots,e_K]$; e_k N-element vector of residuals,

X_k the k-th $J_k \times N$ observables matrix, J_k: block size, N: case number, one observable x_{kj} of the k-th blocks is an N-element vector, $j = 1,\ldots J_k$,

$w_k' = (w_{k1},\ldots,w_{kJk})$; $p_k' = (p_{k1},\ldots,p_{kJk})$ J_k-element coefficient vector.

2. Partial Least Squares Equations

a) The partial least squares measurement model to estimate the latent variables in case of the estimation mode A

Initial values: Standardize the observables $\bar{x}_{kj} = 0$; var $(x_{kj}) = 1$, set $w_{kj}^1 = 1$, calculate $l_k^1 = X_k w_k^1$.

Iteration steps: The following two equations have to be performed for one k after another, beginning with the first and using immediately preceding estimation results.

Be l_k^{i-1} estimated from the preceding iteration for $k = 1,\ldots,K$

aa) The Weight Relation: J_k simple regressions to obtain the weights w_k^i

$$x_{kj} = w_{kj}^i \left(\sum_{h \neq k} a_{hk} s_{hk} l_h^{i'}\right) + d_{kj}^i$$

$$\text{with } a_{hk} = a_{kh} = \begin{cases} 1 & \text{there is a path between } l_h, l_k \\ 0 & \text{there is no path between } l_h, l_k \end{cases}$$

$$s_{hk} = \text{sign}\,(r(l_h, l_k))$$

$$l_h^{i'} = \begin{cases} l_h^{i-1} & \text{for } h > k \\ l_h^i & \text{for } h < k \end{cases}$$

bb) Definition Equation to Obtain the Latent Variable l_k^i

$$l_k^i = \lambda_k^i X_k w_k^i \quad \text{with} \quad \lambda_k^i = 1/\text{var}(X_k w_k^i)$$

convergency

if $|w_{kj}^i - w_{kj}^{i-1}| < 1/1000$ for $k = 1, \ldots, K;\ j = 1, \ldots, J_k$

the iterations are finished, $l_k^i := l_k$

The block structure: After convergence, the block structure can be performed: J_k simple regressions

$$x_{kj} = p_{kj} l_j + u_{kj} \quad k = 1, \ldots, K;\ j = 1, \ldots, J_k$$

b) Estimation of the Structural Path Model of Latent Variables

Each equation with non-zero coefficients of the linear model

$$\mathcal{L} = B\mathcal{L} + \mathcal{E}$$

describes an inner relation of the partial least squares path model. In the case of a recursive system, B is estimated with the single equation ordinary least squares approach.

c) Estimation Modes B and C

The only difference in the partial least squares equations for the estimation modes B and C affects the weight relations. If the k-th block is inward-directed, a multiple regression has to be performed to get the weights w_k^i:

$$\sum_{h \neq k} a_{hk} s_{hk} l_h^i = X_k w_k^i + e_k$$

with a_{hk} and s_{hk} as before.

Estimation mode C: there is at least one inward-directed weight relation;
estimation mode B: all the weight relations are inward-directed.

d) Different Weight Schemes in the Weight Relation

In a) and b) the sum of the latent variables in the regression relation is called the block oriented weight scheme. Evaluating the expression

$$\sum_{h<k} b_{kh}^{i} l_{h}^{i} + \sum_{h>k} s_{hk} b_{hk}^{i-1} l_{h}^{i-1} \quad \text{as weighted sum of} \quad l_h \quad \text{for} \quad k = 1, \ldots, K$$

gives the relation oriented weight scheme. The »disadvantage« of this scheme is that in each iteration step the structural path model has to be estimated too (to get the b_{kh}^{i}).
Dropping the sign factor and the b_{kh} gives the unweighted scheme:

$$\sum_{h \neq k} a_{hk} l_h \qquad (a_{hk} \text{ defined as above})$$

e) Structural Equivalence between a Certain LISREL Specification and the Partial Least Squares Specification

LISREL structural model $\begin{bmatrix} 1 & 0 \\ -b_{32} & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} b_{21} \\ b_{21} \end{bmatrix} l_1 + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$	:=:	Partial least square path model of inner relations $l_2 = b_{21} l_1 + e_1$ $l_3 = b_{31} l_1 + b_{32} l_2 + e_2$
measurement model endogeneous variables $\begin{bmatrix} x_{21} \\ x_{22} \\ x_{31} \\ x_{32} \end{bmatrix} = \begin{bmatrix} p_{21} & 0 \\ p_{22} & 0 \\ 0 & p_{31} \\ 0 & p_{32} \end{bmatrix} \begin{bmatrix} l_2 \\ l_3 \end{bmatrix} + \begin{bmatrix} u_{21} \\ u_{22} \\ u_{31} \\ u_{32} \end{bmatrix}$ exogeneous variables $\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix} l_1 + \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix}$	:=:	block structure $x_{kj} = p_{kj} l_k + u_{kj}$ $k = 1, 2, 3$; $j = 1, 2$
estimation: MAXIMUM LIKELIHOOD		iterative procedure with partial ordinary least squares, weight relations

Table 6: Comparison of the Alienation Model in LISREL and PLS Notation (cf. Jöreskog and Sörbom, 1978, p. 25).

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VI. Abstract

Traditional statistical methods require in general extensive assumptions of factors which often cannot be taken for granted. This paper is an introduction to some basic features of Herman Wold's partial least squares approach for estimating multi-scaled path models. This data oriented and distribution free method has been developed especially for those applications in which the problems are complex and prior knowledge is scarce. The method may be regarded as an intermediate procedure between data description and theoretical analysis with exactly specified models.
For illustration purposes, the second part of the paper presents three applications of this method which indicate the broad area which can be covered. The first results are promising.